HEALTH PRODUCTION FUNCTION FOR PREVENTIVE HEALTH PROGRAMS

D. Wibowo and C. Tisdell

Production function studies have mainly been directed at the formal health care sector, in particular hospitals. Health production function relating medical and/or non-medical health inputs to good health, however, have not been intensively investigated. This paper explores the possibility of employing a health production function to examine the relationship between preventive health programs and health status. A review of previous empirical works is presented. When morbidity or mortality is used as a measure of health status, a modification of the usual production function is needed as morbidity or mortality is expected to decline when health input increase. This paper examines six possible function forms, i.e. linear, quadratic, log-linear, reciprocal log-linear, and double log. The paper also considers the use of a health production function to construct isoquants for health status and to estimate the elasticity of production and the elasticity of substitution between health inputs. Some empirical results on the production relationship between morbidity, safe water, and sanitation are presented.

(This paper has been published as Wibowo, D.H. and Tisdell, C.A. (1992), “Health Production Functions For Preventive Health Programs”, Proceedings of the Fourteenth Australian Conference of Health Economists p. 106-133, Faculty of Economics, Commerce and Management Monash University and National Centre for Health Program Evaluation Fairfield Hospital)
1. Introduction

A health production function describes the relationship between combination of health inputs, both medical and non-medical, and resulting health output. It shows how health inputs interact to produce a particular level of health, and how health status changes if health inputs used and their combination change.

The importance of specifying health production functions becomes apparent when one attempts to determine how to allocate limited resources among alternative health input to produce the largest possible increase in health levels. Without specifying a production function, equi-marginal analysis, an important tool for economics optimization, cannot be carried out [see Doll and Orazem(1984)].

Feldstein (1983) argued that one essential type of information required for economic optimization, among others, is empirical information on the marginal effect on health of each of the health programs (p.21). As such information is hardly available, allocation decisions are being made mostly on the basis of the average benefits, instead of the marginal benefits of the programs [Feldstein (1983), Warner and Luce (1982)]. This approach implicitly assumes a linear health production function without a constant-term, which is not always true.

In addition, specifying health production function is also useful when one product a cost-benefit or cost-effectiveness analysis (CBA or CEA) of alternative health programs. Warner and Luce (1982, p.75) point out that:

... regardless of the method chosen, ... identification of inputs and outputs and specification of the linkage between them provides the basis for estimating costs and benefits (or effectiveness) ...

The importance of health production functions become more obvious when the issue of joint production is concerned, i.e. when a single health inputs produce multiple outputs. Finding the most appropriate method for handling the joint production problem is one of the major difficulties in CBA and CEA in the health sector.

This study attempts to review the empirical use of health production functions. A review of previous empirical works, with a special emphasis on those which incorporate non-medical health inputs, is provided. An example from the authors' works in health production function for water supply and sanitation (WSS) is also presented.
2. Non-medical Health Inputs in The Production of Health

Despite an increasing awareness that medical care is but one of the determinants of health status, health production functions incorporating non-medical health have not been intensively investigated. The use of health production function in health economics have mainly been directed at the production of medical care in the hospital sector [Wagstaff (1989)]. Phelps (1992), for example, described the production of health, as the process of transforming medical care, defined as a set of activities designed specifically to restore or augment the stock of health. The contribution of non medical care factors to restore or augment the stock of health is clearly neglected.

A survey by Wagstaff (1989) indicated that a strong tradition in health production functions has yet to develop in British literature. Most British studies on health production functions focused mainly on the relationship between unemployment and health status. See for example Brenner (1979). This study claims that fluctuations in the mortality rate in England and Wales 1936-76 can largely be explained by current and lagged unemployment. Despite its being widely accepted by policy maker, Brenner’s work has been subjected to critical scrutiny by economists and econometricians (e.g. Stern (1983), Wagstaff (1985), Narendranthan et.al. (1985)). There is no convincing that unemployment is a major determinant of morbidity and or mortality [Stern (1983)], that the social costs of unemployment include premature deaths [Wagstaff (1985)] and that unemployment spells increase the probability of future sickness [Narendranthan et.al. (1985)].

Apart from the above debate, the relationship between unemployment and morbidity or mortality is not a simple production function. A number of intermediate variables usually associated with long term unemployment, e.g. income and living environmental condition, may well affect the relationship. In addition, poor health and illness are often a cause of unemployment for an individual, prompting a simultaneous causality between unemployment and health [Stern (1983)].

On the basis of the review of the proceedings of Australian conferences of health economist 1981-1990, the first health in work in health production function in the Australian literature is found to be that of Richardson and Richardson (1981). This study employs four measures of health outputs, i.e. infant mortality, still births, perinatal deaths, and total death. The medical inputs employed in the model are use of medical service and supply of hospital facilities. The non medical inputs employed include the proportion of the low income group, proportion of Aborigines, proportion of urban population, and education level. This study found a linear relationship between the health output measures and the non-medical inputs. A quadratic relationship between the health output measures and the use of medical services including GP service is also reported. This means that, beyond a point, an increase in use of medical services can produce a poorer health status.
As the British literature, the relation between unemployment and health has attracted considerable attention in Australia. A useful review on this topic is found in Richardson (1985) which covers three types of studies, i.e. cross-sectional, longitudinal, and aggregate time series studies.

Other attempts to describe the production relation between non-medical (preventive health) inputs and the health outcome in Australia are found in cost benefits analysis (CBA) studies of several preventive health programs. The studies include the prevention of congenital Rubella syndrome [Owen, et al. (1984)], the effect of hypertension of reduction in sodium intake [Goss (1985)] and the effect of coronary heart disease of a cholesterol check campaign [Segal (1990)]. It is interesting to note the comments of these studies made by Richardson (1984), Doessel (1985), and Goss (1990), respectively, who pointed out the high level of uncertainty faced when estimating the causal connection between health inputs and outputs. Thus, the method for measuring increase in health outputs resulting from the use of both medical and non-medical health inputs remains debatable.

More specific health production studies which attempt to show the importance of non-medical inputs on health status have produced interesting results. Newhouse and Friedlander (1980) found that medical services are less important than non-medical variables, e.g. education, for health. Studies by Berger and Leigh (1989) and Gupta (1990) also indicate the importance of education on health status. Berger and Leigh (1989), using disability, functional limitations, and systolic blood pressures as measures of overall health, further conclude that schooling directly influences health by increasing the efficiency of an individual’s health production.

The effect on child health of inter- and intra-family heterogeneity is studied by Rosenzweig and Wolpin (1988). It was found that inter-family heterogeneity (e.g. income, schooling of mother) and intra-family heterogeneity (e.g. difference among children) affect child health through parental decision behaviour (e.g. allocation of resources for children and breast feeding).

Yamada et al. (1989) developed production functions for neonatal and child mortality. It was found that increased protein and vitamin A consumption result in higher infant and neonatal mortality rates. After deriving the nutrient prices elasticities, Yamada et al. (1989) concluded that increased price of milk and meat can lead to higher infant and neonatal mortality rates. Another interesting study is done by Lopez et al. (1992). This study shows that the death rate from gastro-intestinal cancer, as an indicator of health hazard resulting from pollution, is significantly affected by running water and sewage drainage, two variables representing pollution abatement activities.

Health production studies have also been done in epidemiology by use of the case-control method, e.g. Victoria et al. (1988), Young and Briscoe (1988), Baltazar et al. (1988), Daniels et al. (1990), and Steenland et al. (1990), as well as in demography, e.g. John (1990) and Gupta (1990).
3. The General Health Production Function for a Community

To develop health production functions for a community, indicators such as morbidity, mortality, the infant mortality rate (IMR) or life expectancy can be used as a measured of community health status.

Now let M donate morbidity of either a single disease or a group of disease. Following Grossman (1972) and Wagstaff (1986), morbidity is considered to be a function of preventive health programs ($P_i$), health care service ($C_j$), community environment and habitat ($H_k$), and socioeconomics variables ($E_l$). Thus, $M$ can be written as follows:

$$M = \Gamma (P_i, C_j, H_k, E_l, \mu) \tag{3.1}$$

where:

- $M$ is the morbidity of disease(s)
- $P_i$ is the $i^{th}$ preventive health programs, $i = 1,2,3, \ldots , n$
- $C_j$ is the $j^{th}$ health care service, $j = 1,2,3, \ldots , n$
- $H_k$ is the $k^{th}$ environment and/or habitat indicator in which the community lives, $k = 1,2,3, \ldots , n$
- $E_l$ is the $l^{th}$ socioeconomics indicators, $l = 1,2,3, \ldots , n$
- $\mu$ is the unobserved health stock in the community

Preventive health programs may include immunization, insecticide spraying for vector-borne disease, surveillance for communicable disease, health promotion and education, nutrition improvement, promotion of breast feeding, investment in water supply and sanitation, etc. health care services include variables such as health expenditure, supply and the use of medical services, the level of medical technology, and the use of medicine in the community. Environmental and habitat indicator include variables such as sanitary living conditions, closeness to rivers, rainfall, and geographical features (coastal plains and mountain ranges). Socioeconomic indicators includes per capita income, education level, migration, etc.

Unlike the usual production function in which output normally increases when the quantities of inputs used in the production process increase, in this case the morbidity of disease(s) decrease when the quantities of inputs used in the production of health increases. This also occurs when either overall mortality rate or the IMR is employed as an output variable.

This unique property has several consequences. First, the expected sign of each independent variable is the reverse of those input variables in the conventional production function. For example, in a linear model, the expected sign of the independent variables is negative instead
of positive. Secondly, the marginal productivity of health inputs and the elasticity of production are also negative. To avoid complications, this study adopts absolute values of marginal productivity and the elasticity of production.

4. A Controversy Over The Effect of Water Supply and Sanitation on Health

Walsh and Warren (1979) claim that investment in water supply and sanitation (WSS) reduce child and infant death by only 0-5 per cent (p.971), and thus are less cost-effective compared to an alternative method for diarrhoeal treatment, i.e. Oral Rehydration Therapy (ORT). This study has had two serious consequences. First, it has diverted funds and attention from WSS investment into ORT application [Briscoe (1984), Okun (1988)]. Secondly, it has intensified a debate over ‘how much improvement in health status can be expected from improved WSS facilities’.

One serious flaw of Walsh and Warren’s study is that it uses child death as an output measure to compare the cost-effectiveness of WSS and ORT. In fact, as investment in WSS can prevent the occurrence of diarrhoea, it would be more appropriate to measure their effectiveness by diarrhoea morbidity instead of by child death. See Wibowo and Tisdell (1992) for details.

Several authors have reported the failure of many studies to show significant reductions in diarrhoea incidence as a result of improved WSS facilities [e.g. Levine et al. (1976), Shuval et al. (1981), Huttly et al. (1985)] a review of 67 studies from 28 countries, however, shows that investments in WSS can reduce diarrhoea morbidity and mortality rates by a median of 22 per cent and 21 per cent, respectively [Esrey et al. (1985)], although most of these studies appear to have serious methodological flaws [Blum and Feachem (1983)]. By use of the case control, it is reported that WSS investment can produce a 20, 20 and 24 per cent reduction in diarrhoea incidence in Malawi [Young and Biscoe (1988)] the Philippines [Baltazar et al. (1988)] and Lesotho [Daniels et al. (1990)], respectively. Yet, one may have doubts about the result as there is a possibility that investments in WSS are not efficacious in reducing diarrhoea incidence rate at the 95 per cent confidence interval (CI). In addition, the case-control method does not indicate what per cent increase in WSS coverage (input) is required to produce a given per cent reduction in diarrhoea incidence (output).

5. Morbidity Production Function For Water Supply and Sanitation

Investment in WSS can be regarded as preventive health variables although these investments benefit not only the health but also other sectors, e.g. agriculture and rural small industry. As these investments produce sanitary living environments for communities, they can also be grouped into the environmental intervention variable.
Morbidity of water-borne disease is chosen as the health output measures because the health benefits resulting from investments in WSS would be better manifested by morbidity rather than by mortality [Briscoe (1984), Okun (1984), Doessel and Wibowo (1991)]. As the morbidity associates with diarrhea accounts for 75 per cent of all morbidity from water-borne disease, it is also used as another dependent variable.

5.1. Model Development

In addition to water supply and sanitation variables, a number of other factors may affect morbidity of diarrhoea. These variables include water quality nutritional status, breast feeding behaviour, food hygiene, personal hygiene and diarrhoea education, income per capita and measles immunization. See Wibowo and Tisdell (1992) for details. Unfortunately data inadequacy, both in quality and availability, precluded the inclusion of those variables. This study, therefore, focused on water supply and sanitation as the only independent variables.

Using the general model in equation 3.1, we have $P_1 = $ safe water supply and $P_2 = $ sanitation facilities. Due to data inadequacy, it is assumed that $P_{i=3,4,..., n}$, $C_i$, $H_k$ and $E_l$ are given. The model is then specified by the following general production functions:

\[
\begin{align*}
\text{MWB} &= f \left( \text{WTR}, \text{SAN} \right) \quad (5.1) \\
\text{MDR} &= f \left( \text{WTR}, \text{SAN} \right) \quad (5.2)
\end{align*}
\]

where:

\[
\begin{align*}
\text{MWB} &= \text{morbidity of water-borne disease, i.e. recorded incidences of diarrhoea, cholera, bacillary dysentery, typhoid fever, paratyphoid fever, and hepatitis A, from January to December 1990, per 1000 population.} \\
\text{MDR} &= \text{morbidity of diarrhoea, recorded incidence of diarrhoea, from January to December 1990, per 1000 population.} \\
\text{WTR} &= \text{safe water supply coverage, i.e. proportion of population having access to sanitation (excreta disposal) facilities (per cent).}
\end{align*}
\]

Six basic function, i.e. linear, quadratic, reciprocal, log-linear ($Y=e^{\beta_0-\beta_iX_i}$), reciprocal log-linear ($Y=e^{\beta_0+\beta_i/X_i}$), and double log (Cobb-Douglas) functions were fitted to the data.

5.2. Alternative Functions and Their Properties

Mathematical details of the function mentioned in section 5.1 are now discussed. The general function specified in equation 5.1 is used as an example. The following discussion, however, is also applicable to equation 5.2.

5.2.1. Linear Function
Let $\text{MWB} = \beta_0 + \beta_1 \text{WTR} + \beta_2 \text{SAN} + e$  
\[ e = \text{error term} \tag{5.3} \]

Then the following condition must be satisfied:

\[ \text{WTR}>0, \; \text{SAN}>0, \; \beta_0>0, \; \beta_{i=1,2}<0 \]

The $\beta_{i=1,2}<0$ condition is required to indicate that morbidity of water borne disease (MWB) decreases as WTR and/or SAN increase(s). The parameters, $\beta_i$, represent the marginal productivities of WTR and SAN, i.e. the first-order partial different of MWB. As $\beta$ are constant when WTR and/or SAN changes, the linear function result in constant returns to each unit increment in WTR and/or SAN.

The elasticity of production resulting from this function is not constant at all values of WTR and/or SAN. This elasticity can be derived by simply dividing marginal productivity over average productivity. Thus, the elasticity of production with respect of WTR can be derived as: $\frac{\beta_1}{\beta_0 + \beta_1 \text{WTR} + \beta_2 \text{SAN}}$. Consequently, two different levels of WTR or SAN may exhibit different return to scale (increasing, constant or decreasing return to scale) depending on the value of the elasticity of production.

### 5.2.2. Quadratic Function

For the purpose of simplicity, let us know assume that there is only one independent variables, e.g. WTR. Then:

$\text{MWB} = \beta_0 + \beta_1 \text{WTR} + \beta_2 (\text{WTR})^2 + e$  
\[ (5.4) \]

Since MWB must be equal to or greater than zero when WTR equals zero, then $\beta_0$ must be equal to or greater than zero. As the critical point of the function, i.e. a point denoting the relative maximum or relative minimum value of the function, occurs, if and only if, WTR$>0$, then we can derive the precondition for $\beta_1$ and $\beta_2$ as follows:

The critical point occurs when $d\text{MWB} / d\text{WTR} = \text{MP}_{\text{WTR}} = 0$

then $\beta_1 + 2\beta_2 \text{WTR} = 0$

$\text{WTR}_{cp} = -\frac{\beta_1}{2\beta_2}$  
\[ (5.5) \]

where $\text{WTR}_{cp}$ is the value of WTR at the critical point. Since WTR must be greater than zero at the critical point, the both $\beta_1$ and $\beta_2$ can not be zero.

There are two possibilities for $\beta_1$ and $\beta_2$, i.e. $\beta_1>0$ if and only if $\beta_2<0$, and $\beta_1<0$ if and only if $\beta_2>0$. The first possibility represents a quadratic function in which the critical point is the relative
maximum value of the dependent variable, MWB. The latter represents a quadratic function in which the critical point is the relative minimum value of the dependent variables, MWB.

It is unlikely that MWB increases as WTR increases. Thus, only a part of the quadratic curve is applicable. In the case of the first possibility, i.e. where $\beta_1>0$ and $\beta_2<0$, the curve starts from the critical point, and $WTR_{cp}$ produces the relative maximum value of WMB. In other words $WTR_{cp}$ becomes the minimum value of WTR. Any value of WTR less than $WTR_{cp}$ is not applicable.

The opposite situation happens when $\beta_1<0$ and $\beta_2>0$, in this case $WTR_{cp}$ produces the relative minimum value of MWB and is the maximum value of WTR. Any value of WTR greater than $WTR_{cp}$ is not applicable.

The marginal productivity of WTR is given by the absolute value of $MP_{WTR}$, i.e. $|MP_{WTR}| = \beta_1 + 2\beta_2 WTR$. What are the characteristics of this marginal productivity? To answer this question we may use equation 5.5. this equation always produce a $WTR_{cp}$ which is always positive or zero. In the case of $\beta_1>0$ and $\beta_2<0$, the right hand side of equation 5.5 can be written as $\beta_1/2|\beta_2|$. At any WTR grater than $WTR_{cp}$ we may pick up $WTR_1$ and $WTR_2$, in which $WTR_1$ is less than $WTR_2$.

If $\delta$ is any positive value then:

$$WTR_1 = \frac{\beta_1}{2|\beta_2|} + \delta$$

and the marginal productivity of WTR at $WTR_1$ is given by

$$|MP_{WTR_1}| = 2\beta_2 \delta$$

while $WTR_2 = \frac{\beta_1}{2|\beta_2|} + (\delta + 1)$

and the marginal productivity of WTR at $WTR_2$ is given by

$$|MP_{WTR_2}| = 2\beta_2 (\delta + 1)$$

Since $WTR_1$ is less than $WTR_2$ and $|MP_{WTR_1}| = 2\beta_2 \delta$ is less than $|MP_{WTR_2}| = 2\beta_2 (\delta + 1)$, then the function’s marginal productivity exhibits increasing returns instead of diminishing returns.

In the case of $\beta_1<0$ and $\beta_2>0$, the right hand side of equation 5.5 can be written as $|\beta_1|/2\beta_2$. At any WTR less than $WTR_{cp}$ we may pick up $WTR_3$ and $WTR_4$ in which $WTR_3$ is less than $WTR_4$.

If $\delta$ is any positive value then:
\[ WTR_3 = \frac{\beta_1}{2\beta_2} - (\delta + 1) \]

and the marginal productivity of WTR at WTR_3 is given by
\[ |\text{MP}_{WTR3}| = 2\beta_2(\delta + 1) \]

while \( WTR_4 = \frac{\beta_1}{2\beta_2} - \delta \)

and the marginal productivity of WTR at WTR_4 is given by
\[ |\text{MP}_{WTR4}| = 2\beta_2 \delta \]

Since WTR_3 is less than WTR_4 and \( |\text{MP}_{WTR3}| = 2\beta_2(\delta + 1) \) is greater than \( |\text{MP}_{WTR4}| = 2\beta_2 \delta \), then the function’s marginal productivity exhibits diminishing returns.

The elasticity of production with respect to WTR \( (\xi_{WTR}) \) can be derived from the quotient \( \text{MP/AP} \) where \( \text{MP} \) is marginal productivity and \( \text{AP} \) is average productivity. As in the linear function, the elasticity of production of a quadratic function is not constant at all level of WTR.

### 5.2.3. Reciprocal Function

Let \( \text{MWB} = \beta_0 + \frac{\beta_1}{WTR^m} + \frac{\beta_2}{SAN^n} + e \)  \hspace{1cm} (5.6)

As \( \text{MWB} \) is expected to decline when WTR and/or SAN increase(s), hence the conditions where \( \beta_0 > 0, \beta_1 > 0, \beta_2 > 0, m > 0 \) and \( n > 0 \) are required. The constant term, \( \beta_0 \), does not represent the intersection between MWB and the vertical axis because the curve is not defined at \( \text{WTR} = 0 \) and/or \( \text{SAN} = 0 \). This constant term represent a horizontal asymptote at the point of \( \text{MWB} = \beta_0 \).

The marginal productivity of WTR is given by:
\[ \text{MP}_{WTR} = -\frac{m\beta_1}{WTR^{m+1}} \]  \hspace{1cm} (5.7)

The negative sign in equation 5.7 incidence that \( \text{MWB} \) decreases when WTR increases. As the denominator is equation 5.7 increases when WTR increases, the absolute value of \( \text{MP}_{WTR} \) also declines. This means that the function complies with the law of diminishing return for each unit increment of WTR.

The marginal productivity of SAN has a similar property to that of the marginal productivity of WTR and is given by:
\[ \text{MP}_{SAN} = -\frac{n\beta_2}{SAN^{n+1}} \]  \hspace{1cm} (5.8)
To estimate the elasticity of production with respect to WTR \((\xi_{\text{WTR}})\), we derived an equation for the average productivity of WTR.

\[
AP_{\text{WTR}} = \frac{\beta_0 + (\beta_1/WTR^m + \beta_2\text{SAN}^n)}{WTR}
\]  

(5.9)

Using equation 5.7 and 5.9 we have:

\[
\xi_{\text{WTR}} = \left| \frac{m\beta_1 (\text{SAN})^n}{\beta_0 (WTR)^m (\text{SAN})^n + \beta_1 (\text{SAN})^n + \beta_2 (WTR)^m} \right|
\]  

(5.10)

Meanwhile, the average productivity of SAN is given by:

\[
AP_{\text{SAN}} = \frac{\beta_0 + \beta_1/WTR^m + \beta_2\text{SAN}^n}{\text{SAN}}
\]  

(5.11)

Let \(\xi_{\text{SAN}}\) denote the elasticity of production with respect to sanitation SAN. Using equations 5.8 and 5.11 we have

\[
\xi_{\text{SAN}} = \left| \frac{n\beta_2 (WTR)^m}{\beta_0 (WTR)^m (\text{SAN})^n + \beta_1 (\text{SAN})^n + \beta_2 (WTR)^m} \right|
\]  

(5.12)

Summing \(\xi_{\text{WTR}}\) and \(\xi_{\text{SAN}}\) we have the total elasticity of production, \(\xi\).

5.2.4. Log-Linear Function

Let \(MWB = e^{\beta_0 + \beta_1 WTR + \beta_2 SAN}\)

(5.13)

Since \(e^{\beta_0}\) is always positive regardless of the sign of \(\beta_0\), the \(\beta_0\) can be any positive, zero or negative real number. However, it is necessary that both \(\beta_1\) and \(\beta_2\) are less than zero to ensure that MWB decreases when WTR and/or SAN increase(s). The marginal productivity of WTR is then given by:

\[
MP_{\text{WTR}} = |\beta_1| e^{\beta_1 WTR + \beta_2 SAN}
\]  

(5.14)

and the marginal productivity of SAN is given by

\[
MP_{\text{SAN}} = |\beta_2| e^{\beta_1 WTR + \beta_2 SAN}
\]  

(5.15)

Given the average productivity of SAN is as follows:

\[
AP_{\text{WTR}} = \frac{e^{\beta_0 + \beta_1 WTR + \beta_2 SAN}}{WTR}
\]  

(5.16)

and the average productivity of SAN is as follows:

\[
AP_{\text{SAN}} = \frac{e^{\beta_0 + \beta_1 WTR + \beta_2 SAN}}{\text{SAN}}
\]  

(5.17)

then the elasticity of production with respect to WTR \((\xi_{\text{WTR}})\) is

\[
\xi_{\text{WTR}} = |\beta_1| WTR
\]  

(5.18)
and the elasticity of production with respect to SAN \((\xi_{\text{SAN}})\) is

\[
\xi_{\text{SAN}} = |\beta_2|_{\text{SAN}} \quad (5.19)
\]

The total elasticity of production is given by

\[
\xi = |\beta_1|_{\text{WTR}} + |\beta_2|_{\text{SAN}} \quad (5.20)
\]

\(\xi_{\text{WTR}}, \xi_{\text{SAN}}, \) and \(\xi\) always increase when WTR and/or SAN increase(s). This means that the proportionate change (reduction) in MWB relative to the proportionate change (increment) in WTR and/or SAN increases when the level of WTR and/or SAN increase(s). If WTR and SAN change simultaneously by the same percentage, the production function 5.13 may exhibit decreasing, constant, or increasing return to scale depending on the value of \(\beta_1\) and \(\beta_2\) and the level of WTR and SAN. The higher is the level of WTR and/or SAN, the function is likely to shift from decreasing return to scale to constant return to scale. If the level of WTR and/or SAN proceeds to increase up to a particular point, the function 5.13 may exhibit increasing return to scale.

5.2.5. Reciprocal Log-Linear Function

Let \(\text{MWB} = e^{\beta_0 + (\beta_1/\text{WTR}) + (\beta_2/\text{SAN})}\) \quad (5.21)

Since \(e^{\beta_0}\) is always positive regardless of the sign of \(\beta_0\), the \(\beta_0\) can be any positive, zero or negative real number. When WTR and/or SAN increase(s), MWB declines only if the signs of \(\beta_1\) and \(\beta_2\) are positive.

The marginal productivity of WTR is then given by:

\[
\text{MP}_{\text{WTR}} = -\beta_1 \text{WTR}^{-2} e^{\beta_1/\text{WTR} + (\beta_2/\text{SAN})} \quad (5.22)
\]

and the marginal productivity of SAN is given by

\[
\text{MP}_{\text{SAN}} = -\beta_2 \text{SAN}^{-2} e^{\beta_2/\text{SAN} + (\beta_1/\text{WTR})} \quad (5.23)
\]

Given the average productivity of SAN is as follows:

\[
\text{AP}_{\text{WTR}} = \text{WTR}^{-1} e^{\beta_1/\text{WTR} + (\beta_2/\text{SAN})} \quad (5.24)
\]

and the average productivity of SAN is as follows:

\[
\text{AP}_{\text{SAN}} = \text{SAN}^{-1} e^{\beta_2/\text{SAN} + (\beta_1/\text{WTR})} \quad (5.25)
\]

then the elasticity of production with respect to WTR \((\xi_{\text{WTR}})\) is

\[
\xi_{\text{WTR}} = -\beta_1 \text{WTR}^{-1} \quad (5.26)
\]

and the elasticity of production with respect to SAN \((\xi_{\text{SAN}})\) is

\[
\xi_{\text{SAN}} = -\beta_2 \text{SAN}^{-1} \quad (5.27)
\]
The total elasticity of production is given by
\[ \xi = -\frac{\beta_1}{\text{WTR}} + \frac{\beta_2}{\text{SAN}} \]  
(5.28)

\( \xi_{\text{WTR}}, \xi_{\text{SAN}}, \) and \( \xi \) always decrease when WTR and/or SAN increase(s). This means that the proportionate change (reduction) in MWB relative to the proportionate change (increment) in WTR and/or SAN decreases when the level of WTR and/or SAN increase(s). If WTR and SAN change simultaneously by the same percentage, the production function 5.21 may exhibit decreasing, constant, or increasing return to scale depending on the value of \( \beta_1 \) and \( \beta_2 \) and the level of WTR and SAN. The higher is the level of WTR and/or SAN, the function is more likely to shift from increasing return to scale to constant return to scale. If the level of WTR and/or SAN proceed to increase up to a particular point, the function 5.21 may exhibit increasing return to scale.

### 5.2.6. Double-Log (Cobb-Douglas) Function

Consider the usual Cobb-Douglas function:
\[ \text{MWB} = \beta_0 (\text{WTR})^{\beta_1} (\text{SAN})^{\beta_2} \]
(5.29)

\( \beta_0 \) must be greater than zero because a zero or a negative \( \beta_0 \) results in an unlikely zero or and impossible negative morbidity.

If \( \beta_{i=1,2} > 1 \) then the critical point of the function occurs at WTR=0 and/or SAN=0, and MWB always increases when WTR and/or SAN increase(s). If \( 0 < \beta_{i=1,2} < 1 \), the function has no critical point. MWB increase when WTR and/or SAN increases (s). However, MWB is supposed to decrease when WTR and/or SAN increase(s), thus \( \beta_{i=1,2} \) must be less than zero.

If \( \beta_{i=1,2} < 0 \), the marginal productivity of WTR (MP_{WTR}) is:
\[ \text{MP}_{\text{WTR}} = \beta_0 \beta_1 (\text{WTR})^{\beta_1-1} (\text{SAN})^{\beta_2} \]
which can be written as:
\[ \text{MP}_{\text{WTR}} = \beta_0 \beta_1 \frac{1}{(\text{WTR})^{\beta_1}} (\text{SAN})^{\beta_2} \]
(5.30)

Since \( \beta_1 < 0 \), then MP_{WTR} has a sign. This mean that MWB decreases if WTR increases. Holding SAN constant, we find that, as the denominator is equation 5.30 increases, \( |\text{MP}_{\text{WTR}}| \) decreases when WTR increases. Thus, the function follows the law of diminishing returns.

The marginal productivity of SAN has similar characteristics to that of WTR and is given by:
\[ \text{MP}_{\text{SAN}} = \beta_0 \beta_2 (\text{WTR})^{\beta_1} (\text{SAN})^{\beta_2-1} \]
(5.31)

The average productivity of WTR (AP_{WTR}) is given by:
\[ AP_{\text{WTR}} = \beta_0 (WTR)^{-1} (SAN)^{\beta_2} \]  
while the average productivity of SAN (\( AP_{\text{SAN}} \)) is given by:
\[ AP_{\text{SAN}} = \beta_0 (WTR)^{\beta} (SAN)^{\beta-1} \]  

The elasticity of production with respect to WTR (\( \xi_{\text{WTR}} \)) is given by:
\[ \xi_{\text{WTR}} = \beta_1, \text{ where } \beta_1 < 0 \]  
And the elasticity of production with respect to SAN (\( \xi_{\text{SAN}} \)) is:
\[ \xi_{\text{SAN}} = \beta_2, \text{ where } \beta_2 < 0 \]

We can see from equation 5.35 and 5.34 that \( \xi_{\text{WTR}} \) and \( \xi_{\text{SAN}} \) are constant and equal to \( \beta_1 \) and \( \beta_2 \), respectively. If \( |\beta_1 + \beta_2| \) is greater than one, then the production function 5.29 exhibits increasing returns to scale. If \( |\beta_1 + \beta_2| \) is equal to one, the function exhibits constant returns to scale. If \( |\beta_1 + \beta_2| \) is less than one, the function exhibits decreasing returns to scale.

5.2.7. The Expected Signs of Regression Parameters, \( \beta_i \)

According to the discussion above, the expected signs of the regression parameters, \( \beta_{i=1,2} \), are summarized in table 1. For the quadratic function, there are five \( \beta_1 \) parameters, \( \beta_{i=0,1, \ldots, 4} \), because each of WTR and SAN has two estimator for \( \beta_1 \).

6. Empirical Results

6.1. Data Collection and Econometric Procedures

Data covering the period January-December 1990 were collected during June-July 1991 from 14 district including 194 sub-district in Central Java, Indonesia. Sub-district are used as the unit of the observation. For details, see Wibowo and Tisdell (1992).

The econometrics package employed in this study is SHAZAM [White et al. (1998)]. The ordinary least square (OLS) method was used initially. After plotting the data, it appears that a vertical asymptote right on the Y-axis (which represent morbidity) and a horizontal asymptote close to/ right on the X-axis (which represent WTR or SAN) exist. Thus, regressions without a constant term (\( \beta_0 \)) were also examined.
TABLE 1.
THE EXPECTED SIGNS OF ESTIMATOR $\beta_i$ FOR VARIOUS HEALTH PRODUCTION FUNCTIONS

<table>
<thead>
<tr>
<th>Specification</th>
<th>$\beta_0$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>$\beta_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>(+)</td>
<td>(-)</td>
<td>(-)</td>
<td>N.A.</td>
<td>N.A.</td>
</tr>
<tr>
<td>Quadratic</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>First alternative</td>
<td>(+)</td>
<td>(-)</td>
<td>(+)</td>
<td>(-)</td>
<td>(+)</td>
</tr>
<tr>
<td>or 0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Second alternative</td>
<td>(+)</td>
<td>(+)</td>
<td>(-)</td>
<td>(+)</td>
<td>(-)</td>
</tr>
<tr>
<td>or 0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reciprocal</td>
<td>(+)</td>
<td>(+)</td>
<td>(+)</td>
<td>N.A.</td>
<td>N.A.</td>
</tr>
<tr>
<td>or 0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log-linear</td>
<td>(+)</td>
<td>(-)</td>
<td>(-)</td>
<td>N.A.</td>
<td>N.A.</td>
</tr>
<tr>
<td>or 0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log-linear Reciprocal</td>
<td>(+)</td>
<td>(+)</td>
<td>(+)</td>
<td>N.A.</td>
<td>N.A.</td>
</tr>
<tr>
<td>or 0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Double Log (Cobb-Douglas)</td>
<td>(+)</td>
<td>(-)</td>
<td>(-)</td>
<td>N.A.</td>
<td>N.A.</td>
</tr>
</tbody>
</table>

*The general mathematical form of each specification is as follows:

Liner: $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2$
Quadratic: $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1^2 + \beta_4 X_2^2$
Reciprocal: $Y = \beta_0 + (\beta_1 /X_1) + (\beta_2 /X_2)$
Log Linear: $Y = \exp[\beta_0 + (\beta_1 /X_1) + (\beta_2 /X_2)]$
Log Linear Reciprocal: $Y = \exp[\beta_0 + (\beta_1 /X_1) + (\beta_2 /X_2)]$
Double-log: $Y = \beta_0 X_1^{\beta_1} X_2^{\beta_2}$

N.A. = not available.

Source: Wibowo and Tissdell (1992)

The omission of the constant term result in a different procedure of computing the sum of squares. Unlike the usual sum of squares which is computed from the mean value, in this case the sum of square is computed from zero. Consequently, we do not have the usual coefficient of determination $R^2$ and adjusted $R^2$. Rather, we have a raw moment of $R^2$ given by $(ESS/\sum Y_i^2)$, where ESS denotes the error sum of squares. Unlike the adjusted $R^2$ which allows a trade off between increased $R^2$ and decreased degree of freedom when a variable is added into the model, the raw moment of $R^2$ does not provide such a trade off. Thus, the raw moment of $R^2$ always increases when a new variable is added to the model.

To test for the existence of heteroscedasticity, multiplicative heteroscedasticity (MH) test was applied first. The procedure is to examine if the variance $\sigma_i^2$ is a multiplicative function of the explanatory variables. Details about statistical test for MH and the method of estimating the Generalized Least Square (GLS) estimators when MH exist can be seen from Judge et al. (1988, pp. 365-369) and Judge et al. (1982, pp. 412-420).

For the purpose of comparative model specification, several criteria were used. These criterion are adjusted $R^2$, Generalized Cross Validation (GCV), Hanan and Quinn criterion (HQ), Rice Criterion (RICE), SHIBATA Criterion, Schwarz Criterion (SC), and Akaike Information Criterion.
(AIC) (Ramanathan, 1989). Specification with a higher value of adjusted $R^2$ and a lower value of the other criteria are preferred.

6.2. A Health Production Function of Best Fit

Using OLS estimation it is found that specifications involving a constant term have poor statistical result. Their adjusted $R^2$'s and F-ratios are low, in the logarithmic specifications the values of adjusted $R^2$ are negative indicating that the function are poorly specified. In some cases, e.g. the linear and reciprocal specifications, the sign of the safe water variable is negative while the expected sign for this variable is positive. Details are presented in Appendix 1.

Appendix 2 presents OLS result on these specifications without a constant term. Having more explanatory variables, the quadratic specifications unsurprisingly showed the highest value in term of raw moment of $R^2$. This does not, however, indicate the statistical superiority of the quadratic specifications over the other specifications. As can be seen from appendix 2, the quadratic specifications exhibit higher values of GCV, HQ, RICE, SHIBATA, SC, and AIC than the other specifications. This mean that the reciprocal specifications are statistically better than the quadratics.

The logarithmic specifications, i.e. log-linear, log-linear reciprocal and double log, exhibit very low values in terms of the of GCV, HQ, RICE, SHIBATA, SC, and AIC criteria. But due to difference in scaling, measurement, these low value do not necessarily imply their statistical superiority. The dependent variable of the logarithmic specifications, log MWB and log MDR, are measured in ones or one tenths, while the dependent variables of the other specifications are measures in ten. Consequently, the TSSs of the logarithmic specifications are in thousands, while those of the others are in hundred thousands. See appendix 4.2. the low values of TSSs in turn result in low values for the ESSs, of GCV, HQ, RICE, SHIBATA, SC, and AIC for the logarithmic specifications.

To compare the goodness of fit between the logarithmic specifications and the others, the raw moment of $R^2$ was used. It is clear from Appendix 2 that the logarithmic specifications, because of their lower values in term of the raw moment of $R^2$, are statistically inferior than the other specifications.
TABLE 2.
REGRESSION ANALYSIS OF MORBIDITY OF WATERBORNE DISEASE (MWB) AND MORBIDITY OF DIARRHEA (MDR), RECIPROCAL SPECIFICATIONS.

<table>
<thead>
<tr>
<th></th>
<th>MWB Regression</th>
<th></th>
<th>MDR Regression</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OLS</td>
<td>GLS</td>
<td>OLS</td>
<td>GLS</td>
</tr>
<tr>
<td>Estimated coefficients</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Safe water supply (WTR)</td>
<td>1133.7</td>
<td>1346.6</td>
<td>846.1</td>
<td>938.5</td>
</tr>
<tr>
<td></td>
<td>(9.1175)</td>
<td>(8.6036)</td>
<td>(9.2580)</td>
<td>(8.6273)</td>
</tr>
<tr>
<td>Sanitation (SAN)</td>
<td>79.8</td>
<td>136.1</td>
<td>60.8</td>
<td>11.5</td>
</tr>
<tr>
<td></td>
<td>(1.894)</td>
<td>(2.1970)</td>
<td>(1.9623)</td>
<td>(2.3082)</td>
</tr>
<tr>
<td>Standardized coefficients</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Safe water supply (WTR)</td>
<td>0.491</td>
<td>0.583</td>
<td>0.498</td>
<td>0.552</td>
</tr>
<tr>
<td>Sanitation (SAN)</td>
<td>0.167</td>
<td>0.284</td>
<td>0.173</td>
<td>0.288</td>
</tr>
<tr>
<td>Specification comparisons</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R2</td>
<td>0.54</td>
<td>0.57</td>
<td>0.55</td>
<td>0.57</td>
</tr>
<tr>
<td>F-ratio</td>
<td>111.92</td>
<td>127.93</td>
<td>116.07</td>
<td>126.15</td>
</tr>
<tr>
<td>GCV</td>
<td>671.07</td>
<td>291.02</td>
<td>362.45</td>
<td>197.91</td>
</tr>
<tr>
<td>HQ</td>
<td>680.14</td>
<td>294.99</td>
<td>367.39</td>
<td>200.61</td>
</tr>
<tr>
<td>RICE</td>
<td>671.07</td>
<td>291.05</td>
<td>362.49</td>
<td>197.94</td>
</tr>
<tr>
<td>SHIBATA</td>
<td>670.78</td>
<td>290.93</td>
<td>362.34</td>
<td>197.85</td>
</tr>
<tr>
<td>SC</td>
<td>693.91</td>
<td>300.96</td>
<td>374.83</td>
<td>204.67</td>
</tr>
<tr>
<td>AIC</td>
<td>670.92</td>
<td>290.99</td>
<td>362.42</td>
<td>197.89</td>
</tr>
</tbody>
</table>

OLS = Ordinary Least Square method
GLS = Generalized Least Square method
Figures in parentheses are t-statistics.

Source: Wibowo and Tissdell (1992)

The reciprocal specification are most preferred because they exhibit the lowest values of the of GCV, HQ, RICE, SHIBATA, SC, and AIC criteria. Their raw moment of R2 values are 0.54 and 0.55 for MWB and MDR regressions, respectively. Which are reasonably acceptable for a cross sectional regression. In addition, only the reciprocal functions showed the significance of both the safe water and sanitation variables. The other specifications failed to show the significance of the sanitation variable.

To test the existence of multiplicative heteroscedasticity (MH), it is examined if the logarithm of the error term (e), obtained from the reciprocal equations is any function of safe water and sanitation. The function applied in this test is also reciprocal. Let \( \Omega \) denote the statistics for the MH test. For the MWB equations, \( \Omega \) equals 7.451 which is greater than \( \chi^2(df=2) \) at the 2.5 per cent significance level. It is concluded that MH exists in the MWB equations at the 2.5 per cent significance level.

For the MDR equations, the value of \( \Omega \) is 4.483 which is very close to \( \chi^2(df=2) \) at the 10 per cent significance level. Using the table of probability integrals of the \( \chi^2 \) distribution [Pearson and Hartley (1970), Table 7], \( \Omega \) was shown to be greater than \( \chi^2(df=2) \) at the 11 per cent
significance level. It is concluded that MH exist in the MDR equation. Thus, the GLS method is employed to estimates $\beta_i$ parameters.

Table 2 presents regression results for the reciprocal functions obtained from the OLS and GLS estimation procedures. It is clear that the GLS method produced better statistical results than did the OLS. The raw moment of $R^2$s for the GLS equations are higher while their GCV, HQ, RICE, SHIBATA, SC, and AIC values are lower than those obtained from the OLS equations. Table 2 shows that the OLS results underestimate $\beta_i$.

The preferred health production function for MWB and MDR are as follows:

$$\text{MWB} = \frac{1346.6}{WTR} + \frac{136.1}{SAN}$$

(6.1)

$$R^2 = 0.57 \quad F\text{-ratio} = 127.93$$

and

$$\text{MDR} = \frac{938.5}{WTR} + \frac{101.5}{SAN}$$

(6.2)

$$R^2 = 0.57 \quad F\text{-ratio} = 126.15$$

Note: * significant at $\alpha = 2.5$ percent
** significant at $\alpha = 0.5$ percent


It can seen from equations 6.1 and 6.2 that both safe water coverage (WTR) and sanitation coverage (SAN) are significant regressors for MWB and MDR. WTR and SAN are significant at the 0.5 and 2.5 percent level, respectively. Safe water is shown to be relatively more important than sanitation for MWB and MDR. The standardized coefficients for WTR are about twice those of sanitation. See table 2. the imply that increased safe water coverage can produce a relatively higher reduction in MWB and MDR than increased sanitation coverage.

Now the following question is addressed: how much reduction in morbidity of water-borne disease and diarrhea can be achieved from a given increase in safe water and sanitation coverage? To answer this question we construct isoquant curves and compute the elasticity of production.

6.3. Isoquants of Morbidity and Their Characteristics

To construct isoquants, four morbidity levels are chosen, i.e. the mean value plus 0.5 standard deviation (SD), the mean value, the mean value minus 0.5 SD, and “the best case”. The best case represents the lowest morbidity level that can be achieved if the coverage of save water and sanitation is maximized. The isoquant curves of MWB and MDR are presented in Figure 1 and 2 respectively. The X-axis represent safe water coverage, while the Y-axis represent...
sanitation coverage. The further a curve is from the origin the higher is the health status produces, in other word, the lower is the morbidity level achieved.

A number of important conclusions can be drawn from Figure 1 and 2. First, to achieve a given morbidity level, minimum level of coverage of safe water or sanitation is required. For example, to maintain morbidity of water-borne disease at 31 per mill (the mean value of our data), it is necessary to have safe water coverage at a level of approximately 45 percent, given the sanitation coverage is 100 percent, or to have sanitation coverage at about 8 percent, given that safe water coverage is 100 percent. In Figure 1 these minimum values can be shown by drawing a vertical line from the point of 45 percent on the X-axis, indicating the minimum value for safe water coverage, or by drawing a horizontal line from the point of 8 percent on the Y-axis, indicating the minimum value for sanitation coverage. Table 3 describes the minimum values for safe water and sanitation coverage required to produce the four morbidity levels.

The second important conclusion is that there is a limit to which safe water supply and sanitation interventions only can reduce morbidity of water-borne disease and diarrhea. The most distant isoquant curve from the origin, i.e. 15 per mill level for MWB or 10.5 per mill level for MDR, indicates this limit.

FIGURE 1
ISOQUANT CURVES OF MORBIDITY (WATER-BORNE DISEASE)
Furthermore, if any one input is held constant at the present coverage level, i.e. at the mean value of WTR or SAN, it is impossible to reach the most distant isoquant representing the lowest morbidity level by increasing the coverage of the other variable up to 100 percent. This can be seen in figure 1 and 2 if a vertical line starting from the point of 56 percent on the X-axis, or a horizontal line at the 39 percent level on the Y-axis is drawn. We can see from these Figures that the lines do not reach the most distant isoquant. Points A, B, C, D, and P in the figure illustrate this conclusion.

6.4. Elasticity of Production and Return to Scale

For morbidity of water-borne diseases, the elasticity of production with respect to safe water supply ($\xi_{WTR-MWB}$) is given by the formula:

$$\xi_{WTR-MWB} = \frac{1346.6 \text{ SAN}}{1346.6 \text{ SAN} + 136.1 \text{ WTR}}$$

(6.3)

while the elasticity of production with respect to sanitation $\xi_{SAN-MWB}$ is given by the formula:

$$\xi_{SAN-MWB} = \frac{136.1 \text{ WTR}}{1346.6 \text{ SAN} + 136.1 \text{ WTR}}$$

(6.4)

For morbidity of diarrhoea, the elasticity of production with respect to safe water is given by

$$\xi_{WTR-MDR} = \frac{846.1 \text{ SAN}}{846.1 \text{ SAN} + 60.8 \text{ WTR}}$$

(6.5)
while the elasticity of production with respect to sanitation $\xi_{\text{SAN-MDR}}$ is given by the formula:

$$
\xi_{\text{SAN-MDR}} = \frac{60.8 \text{WTR}}{846.1 \text{SAN} + 60.8 \text{WTR}} \quad (6.6)
$$

Summing up $\xi_{\text{WTR-MWB}}$ (equation 6.3) and $\xi_{\text{SAN-MWB}}$ (equation 6.4) result in the elasticity of production of MWB ($\xi_{\text{MWB}}$) and summing up $\xi_{\text{WTR-MDR}}$ (equation 6.5) and $\xi_{\text{SAN-MDR}}$ (equation 6.6) result in the elasticity of production of MDR ($\xi_{\text{MDR}}$). We can see from these formulae that $\xi_{\text{MWB}}$ and $\xi_{\text{MDR}}$ are always equal to one. Thus, the production function for MWB and MDR in equations 6.1 and 6.2, respectively, exhibit constant return to scale.

### TABLE 3
MINIMUM REQUIREMENT FOR THE COVERAGE OF SAVE WATER (WTR) AND SANITATION (SAN) AT FOUR MORBIDITY LEVELS

<table>
<thead>
<tr>
<th>Morbidity level</th>
<th>% coverage for:</th>
<th>WTR</th>
<th>SAN</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Waterborne disease (MWB)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>41.7/1000 (mean + 0.5 SD)</td>
<td>33</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>31.0/1000 (mean)</td>
<td>45</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>20.2/1000 (mean - 0.5 SD)</td>
<td>71</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>15.0/1000 (best case)</td>
<td>99</td>
<td>89</td>
<td></td>
</tr>
<tr>
<td><strong>Diarrhoea (MDR)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>31.0/1000 (mean + 0.5 SD)</td>
<td>31</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>23.1/1000 (mean)</td>
<td>42</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>15.2/1000 (mean - 0.5 SD)</td>
<td>66</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>10.5/1000 (best case)</td>
<td>99</td>
<td>91</td>
<td></td>
</tr>
</tbody>
</table>


### TABLE 4
EXPECTED REDUCTION IN MORBIDITY IF BOTH INPUTS ARE INCREASED SIMULTANEOUSLY, OR IF ONE INPUT IS HELD CONSTANT

<table>
<thead>
<tr>
<th>% Input increase</th>
<th>% MWB reduction</th>
<th>% MDR reduction</th>
<th>% WTR increase</th>
<th>% MWB reduction</th>
<th>% MDR reduction</th>
<th>% SAN increase</th>
<th>% MWB reduction</th>
<th>% MDR reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>50</td>
<td>33</td>
<td>33</td>
<td>50</td>
<td>29</td>
<td>29</td>
<td>50</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>75</td>
<td>43</td>
<td>43</td>
<td>75</td>
<td>37</td>
<td>37</td>
<td>75</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>100</td>
<td>50</td>
<td>50</td>
<td>100</td>
<td>44</td>
<td>43</td>
<td>100</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>200</td>
<td>67</td>
<td>67</td>
<td>200</td>
<td>58</td>
<td>58</td>
<td>200</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>300</td>
<td>75</td>
<td>75</td>
<td>300</td>
<td>65</td>
<td>65</td>
<td>300</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>400</td>
<td>80</td>
<td>80</td>
<td>400</td>
<td>70</td>
<td>69</td>
<td>400</td>
<td>10</td>
<td>11</td>
</tr>
</tbody>
</table>

The concept of constant return to scale in this study does not have the conventional interpretation, i.e. if all inputs are increased simultaneously by any positive number, \( p \) percent, the output decreases by \( p \) percent. As can be seen from equation 6.1 and 6.2, if both WTR and SAN are multiplied by \( m \), in other words all inputs are increased by \( (m-1)x100 \) percent, the right hand side of the equations is then multiplied by \( 1/m \). Consequently, the left hand side of the equations, i.e. the level of morbidity, is also multiplied by \( 1/m \) resulting in a reduction in the morbidity of \( (1-1/m)x100 \) percent. For example, if the coverage of both safe water and sanitation are simultaneously doubled \( (m=2) \), in other words, a 100 percent increase in all inputs occur, the morbidity of water borne disease and diarrhea will be halved, a decrease in morbidity by 50 percent. This unusual property of the constant return to scale concept result from the use of reciprocal function in this study. Table 4 present details on the potential morbidity reduction under three scenarios i.e. if all inputs increase, if only WTR increases (SAN is held constant), and if only SAN increases (WTR is held constant).

From the formulae 6.3 – 6.6 we can see that \( \xi_{WTR} \) and \( \xi_{SAN} \) for both morbidity of water-borne disease and diarrhea are always less than one. This means that if the coverage of WTR is multiplied by a positive constant \( m \) while the coverage of SAN is held constant, vice versa, morbidity of water-borne disease and diarrhea decline by less than \( (1-1/m)x100 \) percent. For example if WTR is doubled (a 100 percent increase in safe water coverage), the morbidity of water-borne diseases by 44 percent, given that sanitation coverage is constant. On the other hand, if WTR is held constant, a doubled sanitation coverage results in a 6 percent decrease in morbidity of water-borne disease. Table 4 shows more detailed results.

It can be inferred from Table 4 that a given increase in safe water coverage produces a larger reduction in morbidity of water-borne disease and diarrhea than does the same increase in sanitation coverage. This fact supports the previous conclusion that WTR is a relatively more important factor than SAN for decreasing morbidity of water-borne disease and diarrhoea.

6.5. Elasticity of Substitution

Along an isoquant, the elasticity of substitution between safe water and sanitation \((\eta)\) is found to be constant at 0.5 for both the MWB and the MDR production functions. As the production functions exhibit constant returns to scale, \( \eta \) is also constant along all isoquant curves. These fact indicate that there is a low and constant substitutability between safe water and sanitation at any level of morbidity.
7. Discussion

Production function studies have mainly been directed at the formal health care sector. Although the impact on health status of non-medical/preventive health inputs, e.g. health education, becomes increasingly recognized, health production functions relating medical and/or non-medical health inputs to health status have not been intensively investigated.

Health production studies which incorporate non-medical/preventive health inputs have shown interesting results. Increased vitamin A and protein consumption and increased price of milk and meat, for example, can lead to higher infant and neonatal mortality [Yamada et al. (1989)]. In addition, gastro-intestinal cancer is shown to be significantly affected by running water, sewage and drainage [Lopez et al., 1992]. More interestingly, non-medical variables are shown to be more important than medical services for improving health status [Newhouse and Friedlander (1980)].

Most health production studies cited in this paper, however, discontinue their analysis after showing that a health input is a significant determinant of a measure of health status. Further discussions about various aspects of production functions, e.g. isoquant curve and the elasticity of production, would be useful.

This study has provided additional evidence that safe water and sanitation are efficacious in improving health status as reported by other studies [Young and Briscoe(1988), Baltazar et al. (1988), Daniels et al. (1990), Lopez et al. (1992)]. The health production functions which best fit the data are reciprocal functions, and both safe water and sanitation are shown to be significant for morbidity of water-borne disease and diarrhoea.

Several authors suggested that sanitation may be more efficacious than safe water in reducing diarrhea incidence [Esrey et al. (1985)]. These health production functions, however, indicate that safe water is more important than sanitation

The above suggestion does not mean that sanitation investment should be neglected. The reasons are, first, safe water and sanitation have a low substitutability making it relatively difficult to replace one input with another while maintaining the same morbidity level. Secondly, the reduction in morbidity is unlikely to be maximized (in relation to increased investment) if an increase in safe water coverage is not coverage is not accompanied by an increased in sanitation coverage. Finally, if sanitation coverage falls below the minimum level required to achieve a particular targeted morbidity level, then this target would not be achieved even if safe water coverage is increased to one hundred percent. Thus, to minimize the morbidity level, the coverage of safe water and sanitation facilities must both be increased simultaneously.

We estimate morbidity reduction resulting from a given increasing in safe water and/or sanitation coverage. This differs from the case control method [Young and Briscoe (1988),
Baltazar et al. (1988), Daniels et al. (1990) which estimate morbidity reduction resulting from a shift from ‘being not exposed to safe water/sanitation facilities’ to ‘being exposed to such facilities’. Thus, the case control method implies a rise from zero to a hundred percent in coverage, which is not necessarily so in our study.

Our study indicates a larger reduction in morbidity compared to the other studies [Young and Briscoe (1988), Baltazar et al. (1988), Daniels et al. (1990)]. The twenty percent morbidity reduction reported by those studies requires a rise from zero to a hundred percent coverage. In this study, the same reduction would be produced by a twenty-five percent increase in safe water sanitation coverage.

Figure 1 and 2 indicate that if safe water and sanitation coverage re expanded up to the maximum level (i.e. almost equal or equal to a hundred percent coverage), total eradication of water-borne disease and diarrhea is unlikely. Other factors not included in this study such as habitat and socioeconomics factors may influence the incidence of these disease.

References


## APPENDIX 1

### OLS RESULTS FOR SPECIFICATIONS WITH A CONSTANT TERM

<table>
<thead>
<tr>
<th>Specifications</th>
<th>Regression coefficients</th>
<th>WTR2</th>
<th>SAN2</th>
<th>Adjusted R</th>
<th>F-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Constant</td>
<td>WRT</td>
<td>SAN</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dependent variable : M Wolverines AVS</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linear</td>
<td>28.863  (5.6372)</td>
<td>0.09869 (1.2357)</td>
<td>-0.1011 (-1.3722)</td>
<td>0.003</td>
<td>1.290</td>
</tr>
<tr>
<td>Quadratic</td>
<td>30.741  (2.8951)</td>
<td>0.1404 (0.4273)</td>
<td>-0.3086 (-1.1604)</td>
<td>-0.00023 (0.0904)</td>
<td>0.0023</td>
</tr>
<tr>
<td>Reciprocal</td>
<td>32.173  (8.9835)</td>
<td>-113.23 (-1.1513)</td>
<td>22.159 (0.6153)</td>
<td>0.002</td>
<td>1.193</td>
</tr>
<tr>
<td>Log-linear</td>
<td>2.7996 (4.1093)</td>
<td>0.00158 (0.1526)</td>
<td>-0.00232 (-0.2422)</td>
<td>-0.0053</td>
<td>0.491</td>
</tr>
<tr>
<td>Log-linear reciprocal</td>
<td>2.4984 (5.3916)</td>
<td>7.8741 (0.3502)</td>
<td>3.6888 (0.7916)</td>
<td>-0.0072</td>
<td>0.310</td>
</tr>
<tr>
<td>Double log (Cobb-Douglas)</td>
<td>3.6928509 (1.6944)</td>
<td>-0.02107 (-0.0381)</td>
<td>-0.21929 (-0.7355)</td>
<td>-0.0072</td>
<td>0.310</td>
</tr>
<tr>
<td>Dependent variable : M De рождение AVS</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linear</td>
<td>21.375  (5.6659)</td>
<td>0.06468 (1.0991)</td>
<td>-0.05765 (-1.0616)</td>
<td>0.001</td>
<td>1.097</td>
</tr>
<tr>
<td>Quadratic</td>
<td>25.237  (3.2274)</td>
<td>0.01322 (0.0547)</td>
<td>-0.2213 (-1.1301)</td>
<td>0.0005</td>
<td>0.0017</td>
</tr>
<tr>
<td>Reciprocal</td>
<td>23.62   (8.9700)</td>
<td>-69.407 (-1.5433)</td>
<td>18.448 (0.6967)</td>
<td>0.002</td>
<td>1.193</td>
</tr>
<tr>
<td>Log-linear</td>
<td>2.4554 (3.4018)</td>
<td>-0.0011 (-0.0975)</td>
<td>0.0001 (0.0144)</td>
<td>-0.0104</td>
<td>0.007</td>
</tr>
<tr>
<td>Log-linear reciprocal</td>
<td>1.9845 (3.9835)</td>
<td>12.694 (0.5225)</td>
<td>3.9207 (5.0349)</td>
<td>-0.0042</td>
<td>0.596</td>
</tr>
<tr>
<td>Double log (Cobb-Douglas)</td>
<td>3.7059 (2.3288)</td>
<td>-0.16353 (-0.2731)</td>
<td>-0.18805 (-0.5833)</td>
<td>-0.0074</td>
<td>0.291</td>
</tr>
</tbody>
</table>
## APPENDIX 2

### OLS RESULTS FOR SPECIFICATIONS WITHOUT A CONSTANT TERM

<table>
<thead>
<tr>
<th>Specifications</th>
<th>Regression coefficients</th>
<th>Raw moment R</th>
<th>F-ratio</th>
<th>GCV</th>
<th>HQ</th>
<th>RICE</th>
<th>SHIBATA</th>
<th>SC</th>
<th>AIC</th>
<th>TSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>WTR SAN WTR2 SAN2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Dependent variable : MWB</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No constant term</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linear</td>
<td>-0.454 (-8.5991)</td>
<td>0.013 (0.1657)</td>
<td>0.53</td>
<td>106.233</td>
<td>689.889</td>
<td>699.290</td>
<td>689.964</td>
<td>689.671</td>
<td>713.451</td>
<td>689.815</td>
</tr>
<tr>
<td>Quadratic</td>
<td>0.968 (5.8762)</td>
<td>-0.098 (-0.3761)</td>
<td>-0.006 (-4.1958)</td>
<td>0.001 (0.1606)</td>
<td>0.55</td>
<td>57.449</td>
<td>671.691</td>
<td>689.972</td>
<td>671.989</td>
<td>670.847</td>
</tr>
<tr>
<td>Reciprocal</td>
<td>1133.700 (9.1175)</td>
<td>79.840 (1.8940)</td>
<td></td>
<td></td>
<td>0.54</td>
<td>111.919</td>
<td>670.996</td>
<td>680.139</td>
<td>671.069</td>
<td>670.784</td>
</tr>
<tr>
<td>Log-linear</td>
<td>-0.036 (-5.4132)</td>
<td>0.009 (0.9041)</td>
<td></td>
<td></td>
<td>0.46</td>
<td>81.019</td>
<td>8.666</td>
<td>8.784</td>
<td>8.667</td>
<td>8.663</td>
</tr>
<tr>
<td>Double log (Cobb-Douglas)</td>
<td>0.811 (3.1553)</td>
<td>-0.148 (-0.4993)</td>
<td></td>
<td></td>
<td>0.50</td>
<td>95.016</td>
<td>8.031</td>
<td>8.141</td>
<td>8.032</td>
<td>8.029</td>
</tr>
<tr>
<td><strong>Dependent variable : MDR</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No constant term</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linear</td>
<td>-0.328 (-8.4207)</td>
<td>0.027 (0.4727)</td>
<td>0.53</td>
<td>108.342</td>
<td>376.145</td>
<td>381.270</td>
<td>376.186</td>
<td>376.026</td>
<td>388.991</td>
<td>376.105</td>
</tr>
<tr>
<td>Quadratic</td>
<td>0.693 (5.6807)</td>
<td>-0.048 (-0.2512)</td>
<td>-0.004 (-4.0333)</td>
<td>0.000 (0.1337)</td>
<td>0.56</td>
<td>59.362</td>
<td>363.420</td>
<td>373.311</td>
<td>363.581</td>
<td>362.963</td>
</tr>
<tr>
<td>Reciprocal</td>
<td>846.060 (9.2580)</td>
<td>60.794 (1.9623)</td>
<td></td>
<td></td>
<td>0.55</td>
<td>116.066</td>
<td>362.454</td>
<td>367.393</td>
<td>362.493</td>
<td>362.339</td>
</tr>
<tr>
<td>Log-linear</td>
<td>-0.029 (-4.1046)</td>
<td>0.010 (0.9576)</td>
<td></td>
<td></td>
<td>0.35</td>
<td>51.603</td>
<td>9.83948</td>
<td>9.97356</td>
<td>9.84055</td>
<td>9.83636</td>
</tr>
<tr>
<td>Log-linear reciprocal</td>
<td>89.610 (5.9096)</td>
<td>7.479 (1.4548)</td>
<td></td>
<td></td>
<td>0.34</td>
<td>49.549</td>
<td>9.97866</td>
<td>10.1146</td>
<td>9.97974</td>
<td>9.97550</td>
</tr>
<tr>
<td>Double log (Cobb-Douglas)</td>
<td>0.682 (2.4542)</td>
<td>-0.116 (-0.3612)</td>
<td></td>
<td></td>
<td>0.38</td>
<td>58.958</td>
<td>9.37201</td>
<td>9.49972</td>
<td>9.37303</td>
<td>9.36905</td>
</tr>
</tbody>
</table>